Solution.

Quiz 4 (44371)

MATH 2B, CALCULUS, WINTER 2018

Please write your name and student ID number at the back of the paper. No calculators or phones allowed.

Problem 1. Evaluate the following integrals:

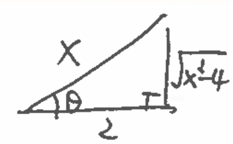
(a). (5 points.) $\int \frac{\sqrt{x^2-4}}{x}\,dx$

Let $x = 2\sec\theta$. then $dx = 2\sec\theta\tan\theta\,d\theta$.

$$\therefore \int \frac{\sqrt{x^2-4}}{x}\,dx = \int \frac{\sqrt{4\sec^2\theta-4}}{\cancel{2\sec\theta}} \cdot \cancel{2\sec\theta}\tan\theta\,d\theta = \int \sqrt{4\tan^2\theta} \cdot \tan\theta\,d\theta = 2\int \tan^2\theta\,d\theta.$$

$$= 2\int (\sec^2\theta - 1)\,d\theta. = 2\tan\theta - 2\theta + C.$$

$$x = 2\sec\theta \Rightarrow \frac{x}{2} = \sec\theta = \frac{1}{\cos\theta} \Rightarrow \cos\theta = \frac{2}{x}.$$



$$\therefore \tan\theta = \frac{\sqrt{x^2-4}}{2}$$

$$\therefore \int \frac{\sqrt{x^2-4}}{x}\,dx = 2\tan\theta - 2\theta + C = 2\cdot\frac{\sqrt{x^2-4}}{2} - 2\arccos\frac{2}{x} + C$$

$$= \sqrt{x^2-4} - 2\arccos\frac{2}{x} + C.$$

(b). (5 points.) $\int x\, sinx\, cos^2x\, dx$ (hint: try integration by parts)

Let $u = x$. $dv = \sin x\cos^2 x\,dx$. then we want to find what's $v(x)$ now.

$v(x) = \int dv = \int \sin x\cos^2 x\,dx$ So if $y = \cos x$. then $dy = -\sin x\,dx$

$$= -\int \cos^2 x\,(-\sin x\,dx)$$

$$= -\int y^2\,dy = -\frac{1}{3}y^3 = -\frac{1}{3}\cos^3 x.$$

So by integration by parts.

$$\int x\sin x\cos^2 x\,dx = x\left(-\frac{1}{3}\cos^3 x\right) + \int \frac{1}{3}\cos^3 x\,dx$$

$$= -\frac{x}{3}\cos^3 x + \frac{1}{3}\int \cos^3 x\,dx = -\frac{x}{3}\cos^3 x + \frac{1}{3}\int \cos^2 x\,(\cos x\,dx). \quad (*)$$

Let $u = \sin x$. then $du = \cos x\,dx$.

$$\therefore (*) = -\frac{x}{3}\cos^3 x + \frac{1}{3}\int (1-u^2)\,du = -\frac{x}{3}\cos^3 x + \frac{1}{3}u - \frac{1}{9}u^3 + C.$$

$$= -\frac{x}{3}\cos^3 x + \frac{1}{3}\sin x - \frac{1}{9}\sin^3 x + C.$$