Solution

Quiz 3 (44372)

MATH 2B, CALCULUS, WINTER 2018

Please write your name and student ID number at the back of the paper. No calculators or phones allowed.

Problem 1.(5 points.) Evaluate

$$\int x \arctan x \, dx$$

(selected from sample final)

Let $u = \arctan x$ and $dv = x\,dx$, which implies $v = \frac{1}{2}x^2$

So by int. by parts. $\int x \arctan x \, dx$

$$= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx = \frac{1}{2}x^2 \arctan x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + C.$$

Problem 2.(5 points.) Evaluate

$$\int \tan^4 x \sec^6 x \, dx$$

$$= \int \tan^4 x \sec^4 x \cdot (\sec^2 x\,dx)$$

$$= \int \tan^4 x \,(1+\tan^2 x)^2 \,(\sec^2 x\,dx).$$ Let $u = \tan x$. $du = \sec^2 x\,dx$

$$= \int u^4 (1+u^2)^2 \, du$$

$$= \int u^4 (1+2u^2+u^4)\, du$$

$$= \int (u^4 + 2u^6 + u^8)\, du = \frac{1}{5}u^5 + \frac{2}{7}u^7 + \frac{1}{9}u^9 + C$$

$$= \frac{1}{5}\tan^5 x + \frac{2}{7}\tan^7 x + \frac{1}{9}\tan^9 x + C.$$