Solution

Quiz 2 (44372)

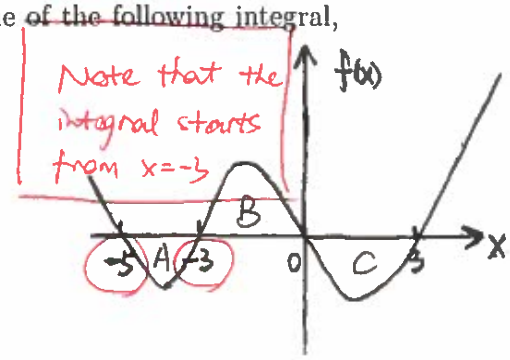
MATH 2B, CALCULUS, WINTER 2018

Please write your name and student ID number at the back of the paper. No calculators or phones allowed.

Problem 1.(5 points.) Each of the regions A, B and C bounded by the graph of f and the $x-$axis has area 3, 4 and 5 respectively. Find the value of the following integral, where $a > 0$ is a constant:

$$\int_{-3}^{3} (|f(x)| + (a+1)\sqrt{9-x^2})\, dx$$

Note that the integral starts from $x=-3$



$I = \int_{-3}^{3} |f(x)|\,dx + (a+1)\int_{-3}^{3} \sqrt{9-x^2}\,dx$

① $\int_{-3}^{3} |f(x)|\,dx = 4+5 = 9$

② $\int_{-3}^{3} \sqrt{9-x^2}\,dx$ represents the area of a semi-circle because if $y=\sqrt{9-x^2}$.

$\Rightarrow x^2+y^2 = 9 = 3^2$. with $\begin{cases} x \in [-3,3] \\ y \geq 0. \end{cases}$

$\therefore \int_{-3}^{3} \sqrt{9-x^2}\,dx = \frac{1}{2}\cdot \pi \cdot 3^2 = \frac{9}{2}\pi$.

$\therefore I = ① + ② = 9 + (a+1)\cdot\frac{9}{2}\pi = \boxed{9 + \frac{9}{2}(a+1)\pi}$

Problem 2.(5 points.) If $f(x) = \int_0^{2x} (1-t^2)e^{t^2}\,dt$, find f' and decide on what interval is f increasing.

Let $u = 2x$. then $f(u) = f(x) = \int_0^u (1-t^2)e^{t^2}\,dt$.

By FTC 1. $f'(x) = \frac{d}{dx}f(x) = \frac{d}{du}f(u)\cdot\frac{du}{dx} = (1-u^2)e^{u^2}\cdot 2$

$= 2(1-(2x)^2)e^{(2x)^2}$

$= 2(1-4x^2)e^{4x^2}$.

Let $f'(x) = 2(1-4x^2)e^{4x^2} \geq 0 \iff 1-4x^2 \geq 0$

$\iff x \in [-\frac{1}{2}, \frac{1}{2}]$