Solution.

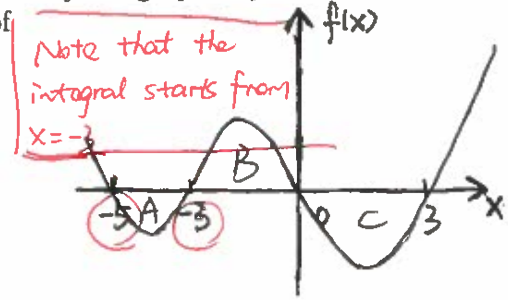
Quiz 2 (44371)

MATH 2B, CALCULUS, WINTER 2018

Please write your name and student ID number at the back of the paper. No calculators or phones allowed.

Problem 1.(6 points.) Each of the regions A, B and C bounded by the graph of f and the $x-$axis has area 3, 4 and 5 respectively. Find the value of

$$\int_{-3}^{3}(f(x)+(x+1)\sqrt{9-x^2})\,dx$$



$$I=\int_{-3}^{3}f(x)dx+\int_{-3}^{3}x\sqrt{9-x^2}\,dx+\int_{-3}^{3}\sqrt{9-x^2}\,dx.$$

① $\int_{-3}^{3}f(x)dx=4+(-5)=-1.$

② If $g(x)=x\sqrt{9-x^2}$. $g(-x)=-x\sqrt{9-(-x)^2}=-x\sqrt{9-x^2}=-g(x)\rightsquigarrow g(x)$ is odd.

$\therefore \int_{-3}^{3}g(x)dx=0$ by the property of odd functions.

③ If $y=\sqrt{9-x^2}$. $\Rightarrow x^2+y^2=9=3^2$. $\begin{cases}x\in[-3,3]\\ y\geq 0\end{cases}$ So the integral $\int_{-3}^{3}\sqrt{9-x^2}dx$ represents the area of a semi-circle

$\therefore \int_{-3}^{3}\sqrt{9-x^2}dx=\frac{1}{2}\pi\cdot 3^2=\frac{9}{2}\pi.$

$\therefore I=①+②+③=-1+0+\frac{9}{2}\pi$

$$\boxed{=\frac{9}{2}\pi-1}$$

Problem 2.(4 points.) $f(x)=\int_{2x}^{3x}\frac{t^2-1}{t^2+1}dt$. Find $f'(x)$.

$$f(x)=\int_{2x}^{3x}\frac{t^2-1}{t^2+1}dt=\int_{2x}^{0}\frac{t^2-1}{t^2+1}dt+\int_{0}^{3x}\frac{t^2-1}{t^2+1}dt$$

$$=\quad g_1(x)\quad+\quad g_2(x).$$

Let $u=2x$ $\frac{du}{dx}=2$. and $g_1(u)=g_1(x)=-\int_{0}^{2x}\frac{t^2-1}{t^2+1}dt=-\int_{0}^{u}\frac{t^2-1}{t^2+1}dt.$

By FTC I. $g_1'(x)=\frac{d}{dx}g_1(x)=\frac{d}{du}g_1(u)\cdot\frac{du}{dx}$

$$=-\frac{u^2-1}{u^2+1}\cdot 2=-2\,\frac{(2x)^2-1}{(2x)^2+1}$$

Let $v=3x$. $\frac{dv}{dx}=3$. and $g_2(v)=g_2(x)=\int_{0}^{3x}\frac{t^2-1}{t^2+1}dt=\int_{0}^{v}\frac{t^2-1}{t^2+1}dt.$

By FTC I. $g_2'(x)=\frac{d}{dx}g_2(x)=\frac{d}{dv}g_2(x)\cdot\frac{dv}{dx}$

$$=\frac{v^2-1}{v^2+1}\cdot 3=3\,\frac{(3x)^2-1}{(3x)^2+1}$$

$$\therefore f'(x)=g_1'(x)+g_2'(x)=-\frac{2(4x^2-1)}{4x^2+1}+\frac{3(9x^2-1)}{9x^2+1}.$$