SOLUTIONS

Math 2B
Winter 2018 44360
Midterm 1
Wed Jan 31 2018
9.00am

Student's Name (Print): ____________________

Student's ID: ____________________

Discussion Section Code: ____________________

Print your name and student ID on the top of this page.

This exam contains 5 pages (including this cover page) and 7 problems. You may *not* use your books, notes, or any calculator in this exam. Do not write in the grading table below.

The following rules apply to the answers you provide in this exam:

- **Organize your work**, in a neat and coherent way.
- **Unsupported answers will not receive full credit.** Calculation or verbal explanation is expected.
- **If you need more space, use the back of the pages**; clearly indicate when you have done this.
- **Box your final answer** for full credit.

Question	Points	Score
1	15	
2	15	
3	5	
4	5	
5	5	
6	5	
7	10	
Total:	60	

1. (a) (5 points) Estimate the area under the parabola $y = x^2$ from $x = 0$ to $x = 4$ using 4 approximating (Riemann) rectangles and right endpoints.

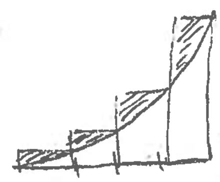
$1(1)^2 + 1(2)^2 + 1(3)^2 + 1(4)^2$

$= 1 + 4 + 9 + 16$

$= 30.$

(b) (5 points) Is this an upper bound or lower bound on the actual area? ~~Why?~~ Illustrate why.

Upper bound:



area under rectangles exceeds area under curve by an amount equal to area of shaded region.

(c) (5 points) Using right endpoints, find an expression for the actual area as the limit of a Riemann sum. Do not evaluate your expression.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\left(i\cdot\frac{4}{n}\right)^2$$

2. Evaluate the following:

(a) (5 points)

$$\int (2 + \tan^2\theta)\, d\theta \quad \text{[Hint: } \tfrac{d}{d\theta}\tan\theta = \tfrac{1}{\cos^2\theta}\text{.]}$$

$$\int (1 + (1+\tan^2\theta))\, d\theta = \int \left(1 + \frac{1}{\cos^2\theta}\right) d\theta$$

$$= \theta + \tan\theta + C$$

(b) (5 points)

$$\int x^3\sqrt{x^2+1}\, dx$$

$u = x^2 + 1$

$du = 2x\,dx$

$x^2 = u - 1$

$$\int (u-1)\cdot\tfrac{1}{2}du\cdot\sqrt{u} = \tfrac{1}{2}\int (u^{3/2} - u^{1/2})\,du$$

$$= \tfrac{1}{2}\left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right] + C = \tfrac{1}{5}(x^2+1)^{5/2} - \tfrac{1}{3}(x^2+1)^{3/2} + C$$

(c) (5 points)

$$\int \frac{\cos(\ln t)}{t}\, dt$$

$u = \ln t$

$du = \frac{dt}{t}$

$$\int \cos u \cdot du = \sin u + C = \sin(\ln t) + C.$$

3. (5 points) Given that $\int_0^9 f(x)\,dx = 4$, evaluate $\int_0^3 x f(x^2)\,dx$.

$u = x^2$

$du = 2x\,dx$

$x=0 \Rightarrow u=0$

$x=3 \Rightarrow u=9$

$$\int_0^9 \tfrac{1}{2}du \cdot f(u) = \tfrac{1}{2}\cdot 4 = 2$$

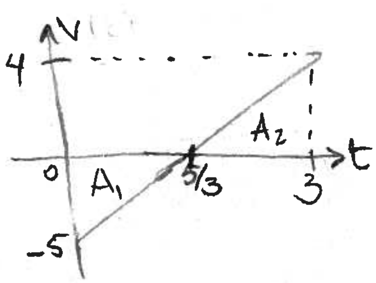
4. (5 points) Evaluate

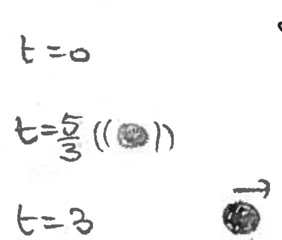
$$\frac{d}{dx}\int_1^{e^x} \ln t\,dt.$$

$u = e^x \Rightarrow \frac{d}{dx}\int_1^u \ln t\,dt = \frac{d}{du}\int_1^u \ln t\,dt \cdot \frac{du}{dx}$

$= \ln u \cdot e^x = xe^x$

5. (5 points) A particle moves along a line with velocity $v(t) = 3t - 5$ at time t. Find the displacement of the object in the time interval $[0, 3]$.

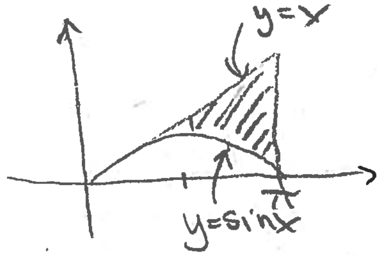




displacement $= s(3) - s(0) = \int_0^3 s'(t)\,dt = \int_0^3 v(t)\,dt.$

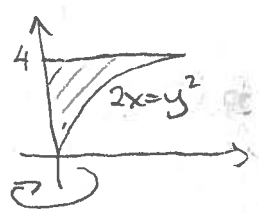
$= -A_1 + A_2 = -\frac{1}{2}\cdot\frac{5}{3}\cdot 5 + \frac{1}{2}\cdot\frac{4}{3}\cdot 4 = \frac{1}{6}(16-25) = -\frac{9}{6} = -\frac{3}{2}$

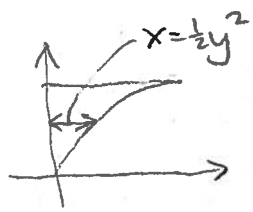
6. (5 points) Compute the area of the region enclosed by $y = \sin x, y = x, x = \pi/2$ and $x = \pi$.



$$\int_{\pi/2}^{\pi} (x - \sin x)\,dx = \left[\frac{x^2}{2} + \cos x\right]_{\pi/2}^{\pi}$$
$$= \left(\frac{\pi^2}{2} - 1\right) - \left(\frac{\pi^2}{8} + 0\right)$$
$$= \frac{3\pi^2}{8} - 1.$$

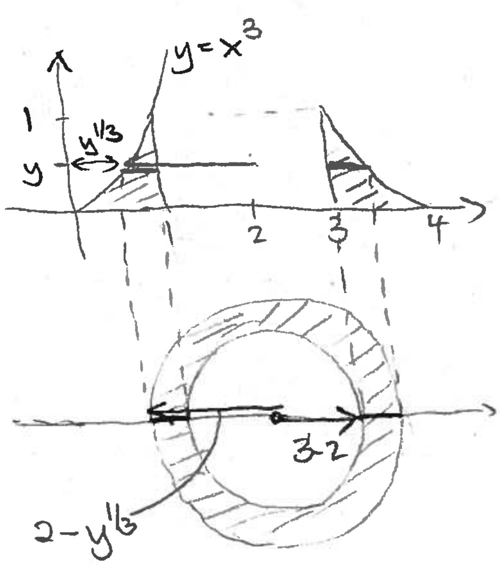
7. (a) (5 points) Find the volume of the solid obtained by rotating the region bounded by the curves $2x = y^2, x = 0$ and $y = 4$ about the y-axis.





$$\int_0^4 dy \cdot \pi \left(\tfrac{1}{2}y^2\right)^2 = \frac{\pi}{4}\int_0^4 dy \cdot y^4$$
$$= \frac{\pi}{4}\frac{y^5}{5}\Big]_0^4 = \frac{\pi}{4}\cdot\frac{4^5}{5} = \frac{\pi}{5}4^4.$$

(b) (5 points) Set up an integral to find the volume of the solid obtained by rotating the region bounded by $y = x^3, y = 0$ and $x = 1$ about the axis $x = 2$. Do not evaluate the integral.



$$\int_0^1 dy\, \pi\left[(2 - y^{1/3})^2 - (3-2)^2\right]$$
$$= \int_0^1 dy\, \pi\left[(2 - y^{1/3})^2 - 1\right].$$