Quiz 2

MATH 175, COMBINATORICS, 25 JAN, 2018

STUDENT ID:
NAME:

Please show all of your work, as partial credit may be given. Please give your explanations in complete sentences. You may use the back of the pages if you need more space.

Problem 1. If S is a multiset, then an *r-combination* of S is an unordered selection of r of the objects of S. Consider the multiset

$$S = \{n \cdot a, 1, 2, 3, \ldots, n\}$$

where the notation $n \cdot a$ means that S contains n copies of a. Determine the number of its n-combiniations; that is, count how many sub-multisets of S have size n. Explain your answer.

For each n-combination, we count the multiplicity of a.
There is $1 = \binom{n}{0}$ n-combinations containing no a's.
There are $n = \binom{n}{1}$ n-combinations containing 1 a.
and so on,
There is $1 = \binom{n}{n}$ n-combinations containing all a's.
we add to obtain the total:
$\sum_{i=0}^{n} \binom{n}{i} = 2^n$ by a previous result.

Problem 2. Prove that

$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}.$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n}{k} \cdot \frac{(n-1)!}{(k-1)!(n-k)!} = \frac{n}{k} \cdot \frac{(n-1)!}{(k-1)!(n-1-(k-1))!} =$$

$$= \frac{n}{k}\binom{n-1}{k-1},$$

as needed.